

THE LOGARITHMIC DERIVATIVE OF THE F_2 STRUCTURE FUNCTION AND SATURATION

E. GOTSMAN

*School of Physics and Astronomy, Tel Aviv University,
Tel Aviv 69978, Israel
E-mail: gotsman@post.tau.ac.il*

We show that when screening corrections are included $\frac{\partial F_2(x, Q^2)}{\partial \ln(Q^2/Q_0^2)}$ is consistent with the behaviour that one expects in pQCD. Screening corrections explain the enigma of the Caldwell plot. Saturation has not been reached at present HERA energies.

1 Introduction

The Caldwell plot¹ of $\frac{\partial F_2(x, Q^2)}{\partial \ln(Q^2/Q_0^2)}$ presented at the Desy Workshop in November 1997 suprized the community. The results appeared to indicate that we have reached a region in the x and Q^2 where pQCD was no longer valid. DGLAP evolution leads us to expect that $\frac{\partial F_2(x, Q^2)}{\partial \ln(Q^2/Q_0^2)}$ at fixed Q^2 would be a monotonic increasing function of $\frac{1}{x}$, whereas a superficial glance at the data suggests that the logarithmic derivative of F_2 deviates from the expected pQCD behaviour, and has a turnover in the region of $2 \leq Q^2 \leq 4 \text{ GeV}^2$ (see fig.1 where the ZEUS data and the GRV'94 predictions are shown). Opinions were voiced that the phenomena was connected with the transition from "hard" to "soft" interactions. Others² felt that the "turnover" in $\frac{\partial F_2(x, Q^2)}{\partial \ln(Q^2/Q_0^2)}$ may be an indication of saturation of the parton distributions.

Amongst the problems that one faces in attempting to comprehend the data, is the fact that due to kinematic constraints the data is sparse, and each point shown pertains to a different pair of values of x and Q^2 . We miss the luxury of having measurements at several different values of x for fixed values of Q^2 , which would allow one to deduce the detailed behaviour of $\frac{\partial F_2(x, Q^2)}{\partial \ln(Q^2/Q_0^2)}$. Bartels et al³ had previously suggested that the logarithmic derivative of the structure function F_2 should be sensitive to screening effects.

2 QCD and $\frac{\partial F_2(x, Q^2)}{\partial \ln(Q^2/Q_0^2)}$

The DGLAP evolution equations⁴ imply the relation

$$\frac{\partial F_2(x, Q^2)}{\partial \ln(Q^2/Q_0^2)} = \frac{2\alpha_s}{9\pi} xG^{DGLAP}(x, Q^2) \quad (1)$$

where $xG^{DGLAP}(x, Q^2)$ denotes the distribution of gluons in the proton. At present HERA energies $xG(x, Q^2)$ grows rapidly with increasing $\frac{1}{x}$ i.e. energy. From unitarity constraints we know that this growth must taper off, and at some value of x ($=x_{cr}$), $xG(x, Q^2)$ must become saturated, and perturbative QCD will no longer be valid.

To illustrate the effects that we can expect for the saturated case, we turn to the colour dipole picture of DIS². Here the γ^* fluctuates into a $q\bar{q}$ pair, which then scatters on the proton over a relatively short time scale compared to the fluctuations. We have

$$\frac{\partial F_2(x, Q^2)}{\partial \ln(Q^2/Q_0^2)} \sim Q^2 \sigma_{q\bar{q}}(\Delta r_\perp \sim \frac{1}{Q}) \quad (2)$$

where $\sigma_{q\bar{q}}$ denotes the cross section for the $q\bar{q}$ pair to interact with the proton, and Δr_\perp the distance between the q and \bar{q} . When the distribution of gluons in the proton is normal pQCD is applicable, and the relation given in eq.(1) holds. However, when the gluons are densely packed (i.e. saturated) one reaches the unitarity limit and the colour dipole cross section can be assumed to be geometric i.e. $\sigma_{q\bar{q}} \sim \pi R_p^2$.

For the saturated case we then expect

$$\frac{\partial F_2(x, Q^2)}{\partial \ln(Q^2/Q_0^2)} \sim Q^2 \pi R_p^2 \quad (3)$$

i.e. the logarithmic slope should grow linearly with Q^2 .

3 Results

We show that the Caldwell plot is in agreement with the pQCD expectations, once screening corrections (SC) (which become more important as one goes to lower values of x and Q^2), are included. To provide a check of our calculations, we compare with the results one derives using the ALLM'97 parametrization⁵, which we use as a "pseudo data base". This parameterization is based on a Regge-type approach formulated so as to be compatible with pQCD and the DGLAP evolution equations.

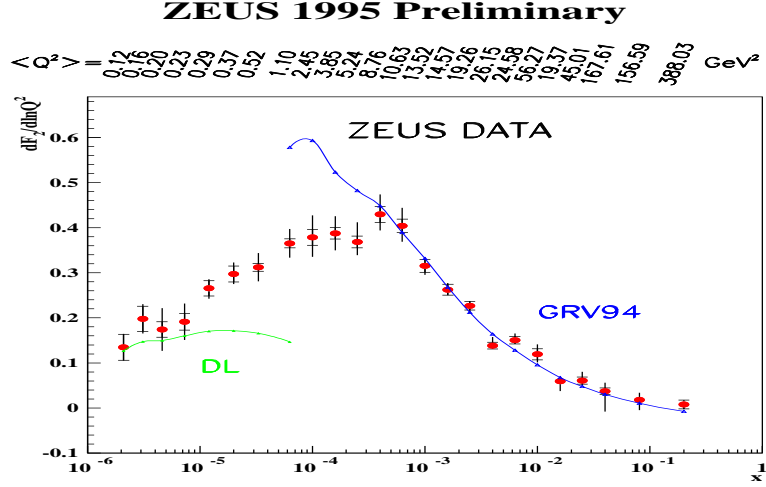


Figure 1: ZEUS data and GRV'94 predictions for F_2 slope

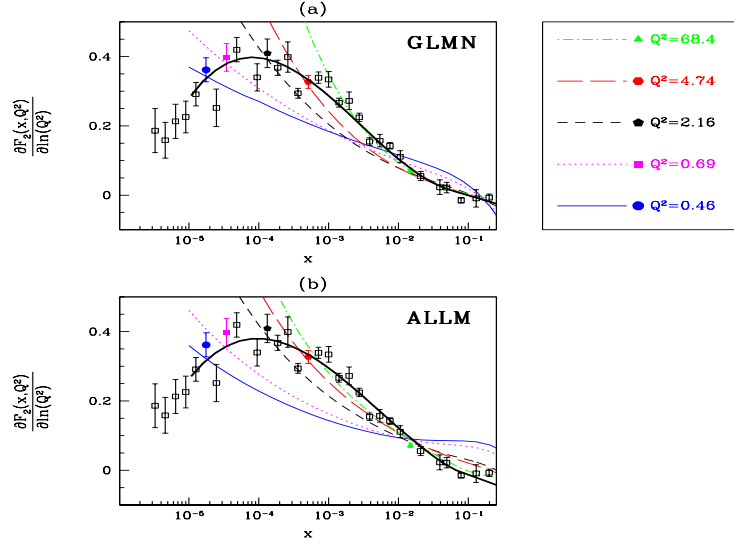


Figure 2: The F_2 slope (a) in our QCD calculation incorporating SC, and (b) in the ALLM'97 parametrization.

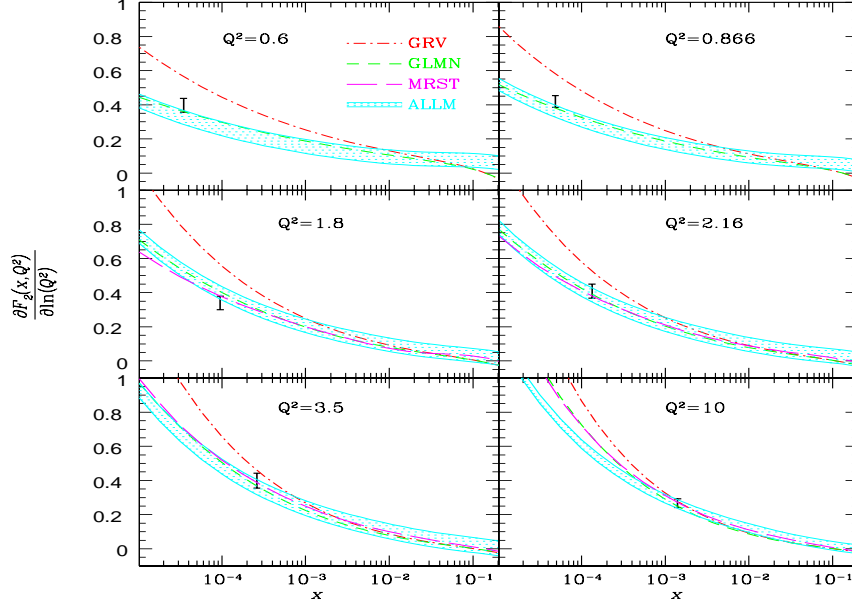


Figure 3: $\frac{\partial F_2(x, Q^2)}{\partial \ln(Q^2/Q_0^2)}$. In addition to the ALLM band we show a typical data point with its error.

Following the method suggested by Levin and Ryskin⁶ and Mueller⁷ we calculate the SC pertaining to $\frac{\partial F_2(x, Q^2)}{\partial \ln(Q^2/Q_0^2)}$ for both the quark and gluon sector. In fig.2 we show the results as well as those of ALLM compared with the experimental results.

In fig.3 and 4 we display our calculations for the logarithmic derivative of F_2 after SC have been incorporated, as well as the ALLM results. In fig.3 for fixed values of Q^2 and varying values of x , and in fig.4 for fixed x and varying values of Q^2 . In fig.4 we show our results as well as those of ALLM compared with the experimental results. We note that $\frac{\partial F_2(x, Q^2)}{\partial \ln(Q^2/Q_0^2)}$ at fixed Q^2 both in our calculations and in the "psuedo data" (ALLM), remains a **monotonic** increasing function of $\frac{1}{x}$.

From fig.4 we note that for fixed x , $\frac{\partial F_2(x, Q^2)}{\partial \ln(Q^2/Q_0^2)}$ decreases as Q^2 becomes smaller. The decrease becomes stronger as we go to lower values of x . This

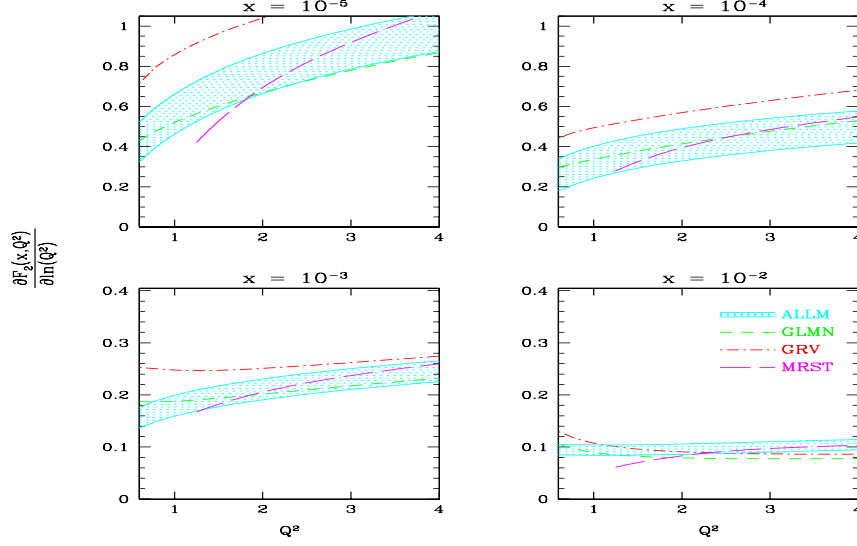


Figure 4: $\frac{\partial F_2(x, Q^2)}{\partial \ln(Q^2/Q_0^2)}$ at fixed x .

phenomena which is due to SC adds to the confusion in interpreting the Cadwell plot.

4 Conclusions

- 1) We have obtained a good description of $\frac{\partial F_2(x, Q^2)}{\partial \ln(Q^2/Q_0^2)}$ for $x \leq 0.1$.
- 2) Our results suggest that there is a smooth transition between the "soft" and "hard" processes.
- 3) SC are essential for describing the Caldwell plot even at present HERA energies where we are far below the saturation region (as $\frac{\partial F_2(x, Q^2)}{\partial \ln(Q^2/Q_0^2)} < F_2(x, Q^2)$). In the saturation region ($x \leq x_{cr}$) we expect $\frac{\partial F_2(x, Q^2)}{\partial \ln(Q^2/Q_0^2)} = F_2(x, Q^2)$.
- 4) At fixed x and/or fixed Q^2 , SC to do not change the qualitative behaviour of $\frac{\partial F_2(x, Q^2)}{\partial \ln(Q^2/Q_0^2)}$, they only produce a smaller value of the slope.
- 5) The apparent turn over of $\frac{\partial F_2(x, Q^2)}{\partial \ln(Q^2/Q_0^2)}$ is an illusion, created by the experimental limitation in measuring the logarithmic derivative of F_2 at particular correlated values of Q^2 and x .

6) Direct experimental evidence supporting our hypothesis, was presented recently by Max Klein at the LP99 conference⁸. He concluded "H1 see no departure from the rising behaviour of $\frac{\partial F_2(x, Q^2)}{\partial \ln(Q^2/Q_0^2)}$ as a function of increasing $\frac{1}{x}$ for $Q^2 \geq 3 \text{ GeV}^2$ ", (this is the lowest value of Q^2 for which H1 presented data).

The detailed calculations and results that this talk was based on, appear in⁹ and¹⁰.

5 Acknowledgements

I would like to thank my friends and colleagues Genya Levin and Uri Maor for an enjoyable and fruitful collaboration. I am also indebted to John Dainton for drawing my attention to the new H1 data. This research was supported in part by the Israeli Science Foundation founded by the Israel Academy of Science and Humanities.

References

1. A. Caldwell, Invited talk, DESY Theory Workshop. DESY, October 1997.
2. A.H. Mueller, hep-ph/9904404 and hep-ph/9902302.
3. J. Bartels, K. Charchula and J. Feltesse, Proceedings "Physics at Hera", Ed. W. Buchmueller and G. Ingelman, Hamburg 1991, Vol 1, 193.
4. V.N. Gribov and L.N. Lipatov, Sov. J. Nucl. Phys 15, 438 (1972); L.N. Lipatov, Yad. Fiz. 20, 181 (1974); G. Altarelli and G. Parisi, Nucl. Phys. B 126, 298 (1977); Yu.L. Dokshitzer, Sov. Phys. JETP 46, 641 (1977).
5. H. Abramowicz and A. Levy, DESY 97-251, hep-ph/9712415.
6. E.M. Levin and M.G. Ryskin, Sov. J. Nucl. Phys. 45, 150 (1987).
7. A.H. Mueller, Nucl. Phys. B 335, 115 (1990).
8. M. Klein, "Structure functions in deep inelastic Lepton-Nucleon scattering", plenary talk, Lepton-Photon'99, XIX International Symposium on Lepton and Photon Interactions at High Energies, Stanford University, August 1999.
9. E. Gotsman, E. Levin and U. Maor, Phys. Lett. B 425, 369 (1998).
10. E. Gotsman, E. Levin and U. Maor, Nucl. Phys. B 539, 535 (1999).